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Option Pricing in Periods of Negative or Low- Interest Rates: Case of European Type of Options

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ABSTRACT

The negative interest rates policy that was implemented to help the economy to recover from a recession, pushed the market to revise some pricing models. The Black Scholes model in the current situation where rates are below zero fails to price interest rate options since it only allows positive rates in its formula. Besides the Black Scholes model, the Heston Cox Ingersoll also doesn't allow negative inputs of interest rates. Deloitte in February 2016 introduced a new pricing regime based on inserting a shifting parameter to existing pricing models like the Black Scholes and the SABR models. This work, analyses the performance of the shifted Black Scholes model in the negative rate environment. In this study, we also conduct a comparative study of pricing models, by providing their performance based on speed, complexity, and market applicability. According to empirical findings, the shifted Black Scholes model performs very well in the negative rate environment. Even though finding the right shift parameter and generating the implied volatility can be challenging, the shifted Black Scholes, especially when backed with the Montecarlo simulation, is equipped with enough tools to price interest rate options when rates are below zero.

Keywords*: Shifted Black model, Option pricing, Negative rates, Implied volatility, Stochastic volatility, Interest rates products*

L'évaluation des Options en Période de Taux D'intérêt Négatifs ou Faibles : Cas des Options de Type Européen

RÉSUMÉ

La politique de taux d'intérêt négatifs mise en œuvre pour aider l'économie à sortir de la récession a poussé le marché à revoir certains modèles de tarification. Dans la situation actuelle où les taux sont inférieurs à zéro, le modèle de Black Scholes ne permet pas de déterminer le prix des options sur taux d'intérêt, car il n'autorise que les taux positifs dans sa formule. Outre le modèle de Black Scholes, le modèle de Heston Cox Ingersoll ne permet pas non plus de prendre en compte les taux d'intérêt négatifs. En février 2016, Deloitte a introduit un nouveau régime de tarification basé sur l'insertion d'un paramètre de décalage dans les modèles de tarification existants tels que les modèles Black Scholes et SABR. Ce travail analyse la performance du modèle Black Scholes décalé dans un environnement de taux négatifs. Dans cette étude, nous menons également une étude comparative des modèles de tarification, en fournissant leurs performances basées sur la vitesse, la complexité et l'applicabilité au marché. D'après les résultats empiriques, le modèle de Black Scholes décalé est très performant dans un environnement de taux négatifs. Même si trouver le bon paramètre de décalage et générer *la volatilité implicite peut s'avérer difficile, le modèle de Black Scholes décalé, en particulier lorsqu'il est soutenu par la simulation de Montecarlo, est doté de suffisamment d'outils pour fixer le prix des options sur taux d'intérêt lorsque les taux sont inférieurs à zéro.*

***Mots clés**: Modèle de Black décalé, Évaluation des options, Taux négatifs, Volatilité implicite, Volatilité stochastique, Produits de taux d'intérêt*

INTRODUCTION

Although The recent financial crisis in 2008 revealed the trustworthiness between counterparties as a major concern in financial transactions. The downfall of some financial institutions affected the World's economy. For many, trading became complex or hardly affordable (underpricing, overpricing, credit risk). In order to avoid this chaotic situation bringing a halt to the economy, central banks, especially in Europe, implemented some exceptional measures. Interest rates were lowered in order to make borrowing cheaper (Deloitte, 2016). The idea is to invite investors to borrow money and inject it into the economy. On June 5th, 2014, this process was taken even further, and negative interest rates were observed for the first time in history. Technically a negative rate implies that putting money in a deposit account will result in a loss. Central banks, as a result, punish investors and central banks for holding their cash. This new policy was supposed to inspire investors to bring in new money, which might as a result, contribute to long-term economic growth.

Using negative interest rates has major economic consequences and requires considerable technical implications. Models that help fix prices for derivatives usually require (high) positive risk-free rates, which therefore becomes challenging in the negative rates environment. For example, the cox Ingersoll ross short rate model only works with positive inputs and, as a result, can not be used for negative interest rates. The famous Black Scholes model used primarily in pricing European options strongly works under the assumption that rates can never be negative. The question remains: What derivative pricing model should be used for accurate results in periods of low or negative rates?

Options theory has a long and rich history. The elaboration of the Black Scholes model as a tool of option pricing in 1973 made options trading much more practical. Since that year, traders have widely used the Black Scholes model to price with accuracy options. With the implementation of negative rates, specific models like the Hull and White, the Heston Vasicek, and the Bachelier are regaining their popularity again.

This study will mainly insist on some existing pricing models. As pricing derivatives require a strong understanding of stochastic calculus, the first part reviews the dynamics behind options pricing. From the simple understanding of the Brownian Motion theory to the concept of option pricing under zero-coupon bonds, the first part of this study will provide the necessary pieces of information used in derivative pricing. Along with some python simulations, real-life market examples will be calibrated to get a strong understanding of the theoretical part of this study.

A shift parameter is added to Black's formula for the main problem of pricing under negative rate dynamics. By assuming that the LIBOR rate follows a log-normal process, the concept of shifting a distribution is very similar to the idea of displacement. The process allows moving the probability density along the X-axis to cover the negative territory.

LITERATURE REVIEW

Interest rates are associated with the idea that a lender request a premium for undertaking the risk of lending money; hence, the logical argument that interest rate is modeled to be positive (Haksar and Kopp, 2020). Traditionally, a model that gives a negative interest rate due to pricing was considered inefficient. With the current situation, most developed countries' interest rates follow a negative trend; models like the Hull- White and the Bachelier were overlooked due to their acceptance of negative values, while underlying pricing assets tend to be taken into consideration. The Hull- White model in the classic book of Brigo Mercurio, states that there is a possibility of short rates "r" going to 0. However, such assessment exists only in the Gaussian distribution theory, which has a minor impact on pricing derivatives like options. An alternative model to the Hull- White is the Bachelier model, where interest rates can be negative, as proposed by Louis Bachelier (1900).

In February 2016, Deloitte, prior to the surge of negative interest rates, proposed the shifted Black Scholes, which consists of adding a shift to the traditional Black Scholes to allow negative input of negative interest rates. However, the model does not tackle the volatility observed in the market; hence the use of a shifted SABR model that is almost similar to the shifted Black Scholes but includes necessary features for the implied volatility. Hagan and collaborators introduced the SABR model in 2002. The model owes its popularity to the fact that it incorporates the implied volatility features of the Black Scholes. The derivation of the SABR model implies certain truncations, which leads to some minor errors. An equivalent way to see the breakdown of the SABR model is to price butterfly spreads which, due to the positivity of the convexity of the cap payoff, should remain positive. Despite its considerable errors, the SABR is still preferably used when pricing the vanilla types of options.

The classic Black Scholes model assumes a constant normal volatility parameter. Practitioners agreed to an implied volatility model to tackle the volatility observed in the market. Local volatility dynamics can be extended to calibrate or measure the implied volatility in Black's formula, but they will still lead to erroneous results in risk metrics dynamics. The SABR model proposed by Hagan in 2002 is a two-factor model that follows the Brownian Motion and incorporates some approximative dynamics of the implied volatility observed in the market. The model is unreliable in a negative rate environment unless it has added a shifting parameter. Luuk Hendrick Frankena (2016) analyzed three different solution methods to cope with negative rates while hedging options. The normal Bachelier has a normal distribution and presents a considerable advantage because it does not have to add a shifting parameter to cope with negative rates. The normal SABR model can be used in a negative rate context; the only disadvantage was the assumption of positive probability on significant negative rates. As a solution, Frankena proposed three boundary models for their capacities of modeling rates from the entire real line without introducing some additional parameters. Pricing and hedging European- type options with different expiries dates require accurate calibration. The SABR can only solve this problem by extending the model with dependent parameters.

Agustin Pineda (2017) analyzed 3 derivative pricing models. Among them, arbitrage-free models are suitable for market pricing curves. The shifted SABR model was designated as well-performing among alternative competitors. The models showed outstanding results for both in-sample and strike out-of-sample analysis. However, when dealing with out-of-sampling maturity, the performance seemed to alter.

A study of six models on options traded on the SP500 was done by Bruce Tsoe Jin (2019): the standard Black Scholes model, the black S- Vasicek model, the BS cox Ingersoll model, the standard Heston Hull White model, the Heston- Vasicek model, and the Heston- Cox İngersoll model. The main purpose was to highlight how models perform under low or negative rates and how accurate they could be when it comes to pricing options. Among the analyzed models, those that assume a cox Ingersoll interest rates types do not perform well in negative rates. The simulated prices present more errors than other models without CIR interest rate calibrating models. The standard Black Scholes model had the worst performance because it does not incorporate the volatility present in the market. Plus, it is known to accept positive values unless a shift parameter is added. The standard Heston model and the Heston

Vasicek had the best performance amongst all the studied models. Bruce Tsoe Jin (2019) suggested adding a shift on the CIR model and a Heston stochastic volatility model to give accurate derivatives prices in periods of low-interest rates.

DATA AND METHODOLOGY

In this study, the notoriety of the famous pricing model Black's 76 is preserved. Since the model itsself doesn't recognize negative rates, a shift parameter is assigned to the model in order to enable its capacity of dealing with negative rates. Python is going to be used in order to calibrate and price European type of options (caplets and floorlets for our case).

Price of caplets and floorlets are used for the shifted black model's implementation. The model's calibration is done in Python where numpy for design and matplotlib for maths formulas and graphs design are the first features to be imported for the model's calibration. When calibrating the shifted black model, the first step is to generate the shifted Brownian Motion for a caplet or a floorlet paths. The second step is essentially based on the probability density function where negative value on the axis are recovered by the shift parameter. The last part is for the option price where all the shift parameters follow a same trend to converge on the same price.

The shifted model can also be backed by the exact solution of the montecarlo simulation. Even though its application is not mandatory when pricing, the montecarlo simulation is a good tool for the verification of the exact price of an option.

Pricing derivatives under negative rates: Theory and practice.

a) The Brownian motion.

Derivatives like every other following a Weinar process or a Brownian motion. It's then very important before every pricing process to understand the dynamics behind the Brownian motion.

Let us assume that every time a coin is flipped in the air and lends on heads, a person "x" earns a dollar. When the same coin lends on tale, the same person "x" will lose a dollar. Mathematically it can be represented by: Ri:$\pm\mathbf{1}$ (randomness or changes).

Outcomes will therefore be summed up by Si: $\sum_{\boldsymbol{j=1}}^{\boldsymbol{i}} \boldsymbol{Rj}$ (3.0)

The Brownian Motion also includes two popular properties in probabilities: the Markov property and the martingale property. Both insist on the randomness of a variable. The Markov property insists that past solutions do not impact or influence future solutions. The martingale property states that an expected outcome always equals the present outcome. With the martingale property, our example's expected return or outcome can mathematically be represented by: $\boldsymbol{E}\left[\frac{\boldsymbol{Sj}}{\boldsymbol{Si}}, \boldsymbol{i} < \boldsymbol{j}\right] = \boldsymbol{Si}$. (3.1)

To make the randomness of every solution continuous, we use the Brownian motion. To demonstrate to continuity of the randomness, we divide it into "n" over the time "t."

The Brownian Motion has the following properties:

-E[X(t)]=0 and E=[X(t²)]=t

-The Brownian Motion is continuous

-The Brownian Motion obeys the martingale property: E[X(t)/X(τ), τ<t]= X(τ)

- The Brownian Motion follows a normality relation where τ< t with a mean reversion "0" and a standard deviation $\sqrt{\boldsymbol{t}} * \boldsymbol{\tau}$.

One approach to developing a more realistic model for asset price, which still retains the properties of the random walk as the market efficiency suggests it, is to derive a continuous-time model from random walks. One way to do it is to take the limit as the number of jumps in any unit of time goes to infinity. If this procedure is done, there is necessary to scale down or shrink the size of jumps. For instance, the variance will go to infinity.

Let $\boldsymbol{\sigma^2}$= var (Xj)

Lim var (Sm) = $\lim_{m\to\infty} var\ (\sum_{j=1}^{m} Xj)$ (3.2)

By independence we will have = $\lim_{m\to\infty} \sum_{j=1}^{m} var\ (Xj)$

$$= \lim_{m\to\infty} m\sigma^2 = \infty \ (3.3)$$

The jumps Xj of size 1 must be scaled down for the limit to be sensible. The jumps should reduce by dividing by some factor.

$$\frac{Xj}{\alpha}$$

The question that can arise here is how $\boldsymbol{\alpha}$ should be chosen.

The central limit theorem suggests that:

$\alpha = \sqrt{m}$: $\lim_{m\to\infty} (\sum_{j=1}^{m} \frac{Xj}{\sqrt{m}})$ = $\lim_{m\to\infty} \sum_{j=1}^{m} var\ (\frac{Xj}{\sqrt{m}})$ (3.4)

The probability theory allows us to use: var $(\alpha x) = \alpha^2 var(x)$ (3.5)

By independence, we will then have

$\lim_{m\to\infty} \sum_{j=1}^{m} \frac{1}{m} var\ (Xj)$ = $\lim_{m\to\infty} \frac{1}{m} \sum_{j=1}^{m} \sigma^2$ = $\lim_{m\to\infty} \frac{1}{m} m\sigma^2$ = σ^2 with $\sigma^2 < \infty$ (3.6)

Suggests that $\sqrt{m}$ is the right scaling. Here the purpose is to consider the limit as the number of time steps m per unit gets large. For any real-time t$\geq \mathbf{0}$ the number of jumps will be some integer closer to mt. Scaled random walk is going to be defined by $s_t^{(m)} = \sum_{j=1}^{[mt]} \frac{Xj}{\sqrt{m}}$

Here $[m]$ is the floor function, which is the greatest integer less than X, so for example, $[3,7] = 3$; $[0,9] = 0$.

The Brownian Motion is a continuous-time stochastic process conventionally denoted W(t) with the following properties:

0. W(0)= 0 with probability 1.
1. The sample paths are continuous with probability 1
2. For t$< s < r$ W(s)- W(t) is independent of W(r)- W(s)
3. For any t,s $\geq \mathbf{0}$. W(t)- W(s) is normally distributed with a mean 0

The independence of jumps in the random walk passes to the independence of increments in the limit. The random walk converges to Brownian Motion in an exact sense $s_t^{(m)} \to W(t) as\ m \to \infty$. In the sense of weak convergence of stochastic processes.

The Brownian Motion with a drift

A model should support a trend to be a suitable representation of asset prices. The Brownian Motion is not sufficient because the expectation stays 0 all the time.

In order to remedy this, a term that represents a trend has to be added to the Brownian motion. It is also desirable to add a volatility factor $\boldsymbol{\sigma}$. Brownian Motion with drift is the stochastic process W(t,$\boldsymbol{\mu}, \boldsymbol{\sigma}$)= $\boldsymbol{\mu t} + \boldsymbol{\sigma W(t)}$ (3.7)

$\boldsymbol{\mu t}$ is the drift term, and time allows adjustments of the volatility of the Brownian Motion term (Etienne and Vallois,2007).

b. The Black Scholes Model

The ito's lemma process can help to derive the Black Scholes model and obtain it's PDE equation:

Let's consider S(t) an asset's price and V(s,t) an option price.With the stochastic formula of a stock's price behavior:

dS=μSdt+δSdz, under the following assumptions:

$\pi = a\ position\ taken\ for\ a\ given\ underlying\ asset$

$\boldsymbol{\pi} = \boldsymbol{V} - \Delta\boldsymbol{S}$ (The hedging factor) (3.8)

(4.0) can also be expressed: $\boldsymbol{d\pi} = \boldsymbol{dV} - \Delta\boldsymbol{ds}$. With the multidimensional derivation of Ito's lemma we will have (Chalamandaris& G. Malliaris,2009):

DV= $\frac{\partial V}{\partial S}\boldsymbol{dS} + \frac{\partial V}{\partial t}\boldsymbol{dt} + \frac{1\delta^2 S^2 \partial^2 V}{2\partial S}$ (3.9)

D$\boldsymbol{\pi} = \frac{\partial V}{\partial S}\boldsymbol{dS} + \frac{\partial V}{\partial t}\boldsymbol{dt} + \frac{1\delta^2 S^2 \partial^2 V}{2\partial S} - \Delta\boldsymbol{dS}$ (3.10)

d$\boldsymbol{\pi}$= $\frac{\partial V}{\partial S}\boldsymbol{dS} + \frac{\partial V}{\partial t}\boldsymbol{dt} + \frac{1\delta^2 S^2 \partial^2 V}{\partial S^2}\boldsymbol{dt} - \Delta\boldsymbol{ds}$ (3.11)

d$\boldsymbol{\pi} = \frac{\partial V}{\partial S} - \Delta)\boldsymbol{dS} + (\frac{\partial V}{\partial t} + \frac{1\delta^2 s^2 \partial V}{\partial V^2})\boldsymbol{dt}$ if we choose $\Delta = \frac{\partial V}{\partial S}$ then

d$\boldsymbol{\pi} = (\frac{\partial V}{\partial t} + \frac{1\delta^2 S^2 \partial^2 V}{2\partial S^2})\boldsymbol{dt}$. To avoid arbitrage, the equation is often interpreted this way

d$\boldsymbol{\pi} = \boldsymbol{r\pi dt\ if\ we\ replace\ \pi\ by\ its\ real\ value\ we\ will\ have}$: $\boldsymbol{d\pi} = \boldsymbol{r(v - \Delta S)dt}$ the PDE equation of the Black Scholes will finally be obtained by replacing d$\boldsymbol{\pi}$ by its derivative values:

$\frac{\partial V}{\partial t} + \frac{1}{2}\boldsymbol{\delta}^2\boldsymbol{S}^2\frac{\partial^2 V}{\partial S^2})\boldsymbol{dt} = (\boldsymbol{rV} - \boldsymbol{r}\frac{\partial V}{\partial S}\boldsymbol{S})\boldsymbol{dt}$

= $\frac{\partial V}{\partial t} + \frac{1}{2}\boldsymbol{\delta^2 S^2}^{\frac{\partial^2 V}{\partial S^2}} + \boldsymbol{rS}\frac{\partial V}{\partial S} - \boldsymbol{rV}$ (3.12) (the partial differential equation of the Black Scholes model)

After the transformation of the partial differential equation the black schole model for options pricing can be described as the following for both put and calls:

C =$\boldsymbol{SN(d_1)} - \boldsymbol{N(d_2)K\ e^{-r(T-t)}}$ (3.13)

For puts options: $\boldsymbol{P} = \boldsymbol{N(d_2)ke^{-r(T-t)}} - \boldsymbol{SN(d_1)}$ (3.14)

With:

$\boldsymbol{d_1} = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$. And

$\boldsymbol{d_2} = \boldsymbol{d_1} - \boldsymbol{\sigma\sqrt{T-t}}$

Where:

- S is the stock price.
- K is the option strike price.
- T – t is the time to maturity).
- r is the risk free of interest rate.
- ln is the natural logarithm.
- $\boldsymbol{\sigma}$ the volatilityof the stock.

For negative rates pricing the simple Black Scholes model can't be used since it doesn't allow negative inputs of interest rates unless a shift parameter is added to the model.

The shifted Black Scholes model.

In this work, the shifted Black is proposed as the alternative solution to pricing options under negative interest rates. The application case will be done on caplets and floolrets prices with some set of data collected from Thomson Reuters.

*** Pricing caplets under negative rates**

Caplets or floorlets can be priced under negative rate evironment by an adaptation of the underlying dynamics of the libor rate. The process consist in adding a shift parameter to the libor rate in the case of a European option we add a shift parameter to the Euribor observed on the market. The shift added to the libor rate is defined the following way:

$\hat{\boldsymbol{l}}(\boldsymbol{t}) = \boldsymbol{l_k}(\boldsymbol{t}) + \boldsymbol{\theta_k}$ (4.0)

The process of the libor rate is governed by the log normal process under the following Dynamics:

$\boldsymbol{d\hat{l}}(\boldsymbol{t}; \boldsymbol{T_k} - \boldsymbol{1}, \boldsymbol{T_k}) = \widehat{\boldsymbol{\sigma_k}}\, \hat{\boldsymbol{l}}(\boldsymbol{t}; \boldsymbol{T_k} - \boldsymbol{1}, \boldsymbol{T_k}) \boldsymbol{dW_k^K}(\boldsymbol{t})$. (4.1)

Shifting the process simply means moving the probability density along the X-axis. The shift process requires some sort of the delicacy when choosing a shift parameter. A pricing expert have to make sure that the chosen shift is closer to zero. To make sure of the convrgence, one can always back up the pricing formula with the exact simulation of the montecarlo although it's not necessary all the time.

The pricing formula with the shift parameter of a caplet is given by: $\boldsymbol{V_k^{cpl}}(\boldsymbol{t_0}) = \boldsymbol{N_k T_k P}(\boldsymbol{t_0}, \boldsymbol{T_k})[\widehat{\boldsymbol{l_k}}\,(\boldsymbol{t_0})\boldsymbol{N}(\boldsymbol{d_1}) - \widehat{\boldsymbol{K}}_{\boldsymbol{k}}\boldsymbol{N}(\boldsymbol{d_2})$ (4.2) with

$$\boldsymbol{d_1} = \frac{\log\left(\frac{l(t_0;T_k-1,T_k)}{K}\right)+\frac{1}{2}\sigma^2 K(T_k-t_0)}{\sigma_k(\sqrt{T_k-t_0}}$$ and

$$\boldsymbol{d_2} = \boldsymbol{d_1} - \boldsymbol{\sigma K}\sqrt{\boldsymbol{T_k} - \boldsymbol{t_0}}$$

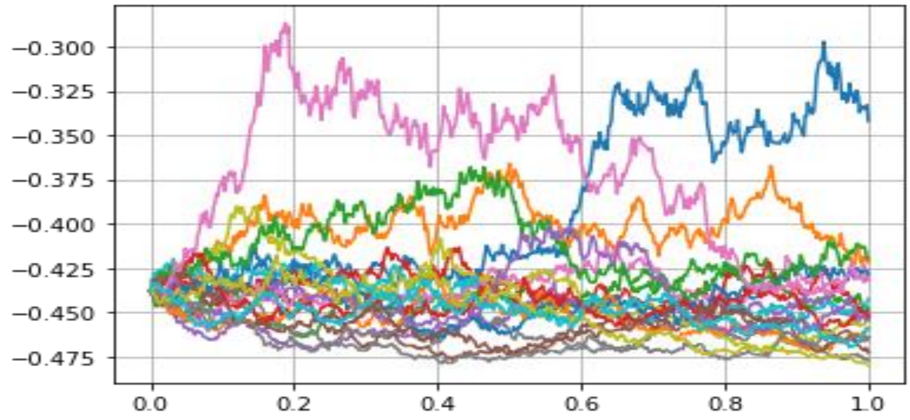
Shifting parameters are included in the libor rate with$\widehat{\boldsymbol{K}} = \boldsymbol{K} + \boldsymbol{\theta_k}$ and$\widehat{\boldsymbol{l_k}}(\boldsymbol{t_0}) = \boldsymbol{l_k}(\boldsymbol{t_0}) + \boldsymbol{\theta_k}$ (4.3)

EMPIRICAL FINDINGS

For an application example we first simulate an option of a one year maturity with an Euribor of 3M at-0.461 the shift parameter is 0.5, the strike price is 0.445 the log normal volatility is 0.75% we get the following results step by step

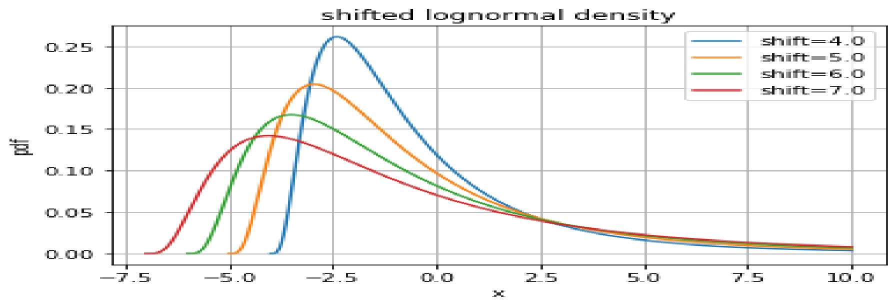
- The shifted brownian motion:

Figure 1: Caplets paths under the Brownian motion



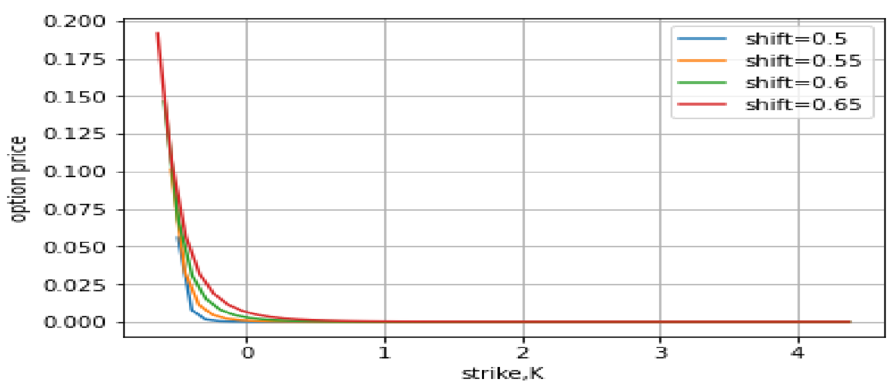
- The shifted log normal density:

Figure 2: Recovering negative values with the shift parameter



- The real option price

Figure 3: Real options prices



Shift parameters determines the lowest level of negative interest rates. However due to constant change of the Euribor, the shift parameter has to be adjusted frequently. The challenging part about the shifted Black model is that it has to be manipulated with precaution. The shift parameter must not be very bigger than the actual libor rate. It has to be the closest possible. Due to that argument of the right shift parameter to use, the market convention often provides the informtions on the right shift parameter to use.

The shift parameter is always associated with the time to maturity of the underlying. The shifted black model is slightly different from the classic Black Scholes model. In order to prevent negative rates to impact the strike price the shift parameter is also added to the strike parameter. The strike price becomes $\widehat{\boldsymbol{K}} = \boldsymbol{K} + \boldsymbol{\theta_k}$ under the new libor $\hat{\boldsymbol{l}}_{\boldsymbol{k}}(\boldsymbol{t_0}) = \boldsymbol{l_k}(\boldsymbol{t_0}) + \boldsymbol{\theta_k}$

The only problem with the shifted black model, the implied volatility has to be found which may lead some practitioner to prefer the shifted SABR model over the shifted Black model. In practice, when calibrating the shifted Black model, the implied volatility has to be calculated. If a wrong volatility value is used, the option price will be erroneous.

CONCLUSION

Negative rates like stated in the introductive part of this study, are somehow a new concept that was established in order to boost the economy that was recovering from a recession. Denmark was the first to implement negative rate policy in 2012 followed by the European central bank in 2016.

The implementation of negative rates can have both positive and negative impacts. On one hand borrowing becomes cheaper and there’s a lot of cash inflows in the economy. On the other hand rate cuts below zero, implies for commercial bank that there’s no longer cash reserves to hold on since it can result into a money- loss activity. With negative rates, comes the ptoblem of reviewing pricing models like it’s the case for derivatives where classic models like the Black Scholes model where built under the assumption that rates will never become negative.

The derivative world has a lot of pricing models each with different features. In this study, simple European type of options were analysed. The less models are complex, the more they are likely to be used on the market. When it’s question of speed there’s no doubt fast fourier transform models give accurate results. For large and complex numbers the Montecarlo simulation is often preferred.

Choosing a pricing model depends on the type of instrument one wants to price. Among the appropriate models for negative interest rates, the Hull and White model and the SABR model, we choosed to work with shifted Black Scholes model as it seems to be the model with huge stability that can handle any type of derivative and with less complexity. The SABR model has a good performance when it comes to the negative rate environement because it can also incorporate a shift parameter in its features. The only problem with that model is it’s complexity but as advised by deloitte (october 2016), the free boundary SABR by Antonov et al seems less complex when dealing with negative rates. The Hull and White model is gaussian distribution type of model. Which implies that there is a probability of rates being negatives. The Gaussian properties of Hull-White 1 factor model is:

$\boldsymbol{p(r(t) \leq x) = \phi(\frac{x - \mathbb{E}(r(t))}{Var(r(t))})}$. The probability of rates becoming negative therefore becomes:

$\boldsymbol{p(r(t) \leq 0 = \phi(-\frac{r(s)e^{-a(t-s)} + \alpha(t) - \alpha(s)e^{-a(t-s)}}{\sqrt{\frac{\alpha}{2a}}(1 - e^{-2a(t-s)})})}$. The Hull and White model is a good model for the derivative pricing in both and positive and negative environement. Its calibration is not fast and is often expensive to use hence its rare application on the market. For this sudy the shifted Black-Scoles model was used to price options under low or negative rates environement. Even though choosing the shift parameter can be challenging, the model is handy and can be very well applied on today’s market.

REFERENCES

Altavilla, B & Gürkaynak, S (2009), Measuring Euro Area Monetary Policy, *European central Bank Working Paper Series*, No 2281. https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2281

Augustin, P. G. (2017). Negative Rates in Derivatives Pricing Theory and Practice, Master Thesis, July. https://www.uv.es/bfc/TFM2017

Bachelier, M. (1900). Théorie de la spéculation. Annales scientifiques de l'ecole normale supérieure, 3(17):21-86.

Barlett ,B. (2006). Hedging under SABRModel. *Gorilla Science,* 42, 1011-1020

Black, F&Scholes, M. (1973). The pricing of Options and Corporate Liabilities. *Journal of Political Economy*, vol81, No.3 pp637-654.

Black, F&ScholesM. (1972). The valuation of Options Contract and Test of Market Efficinency. *The Journal of Finance*. Vol27, No2, Papers and proceedeings of the Thirteen Annual Meeting, pp399-417, https://www.jstor.org/stable

Central Bank Digital Currencies (CBDC). (2021). Financial stability implications. https://www.bis.org/publ/othp42_fin_stab

Chalamandaris, G&Malliaris G.(2009) Ito's Calculus and Derivation of the Black Scholes Option-Pricing Model. *Handbook of quantitative finance, C.F. Lee, AliceC.Lee, springer*

Corporate Finance Institute (2020). Negative Interest Rates, June. https://corporatefinanceinstitute.com/resources/knowledge/finance/negative-interest-rates.

Deloitte (2016). Interest rate derivatives im the negative-rate environement: Pricing with a shift. https://www2.deloitte.com/content/dam/Deloitte/global/Documents/Financial-Services,

Etienne T & Vallois P (2006). Range of Brownian Motion with Drift. Journal of theoretical probability 19(1) pp45-69 10.1007/s10959-006-0012-7

Fang F. (2010). "The COS Method: An Efficient Fourier Method for Pricing Financial Derivatives. *Disertation at Delt University*. https://core.ac.uk

Frankena, L, H. (2016). Pricing and Hedging Options in a Negative Interest Rate Environment. Master Thesis. https://www.semanticscholar.org/paper/Pricing-and-hedging-options-in-a-negative-interest-Frankena

Grzelak, L & Oosterlee, W. (2011). "Calibration and Monte Carlo Pricing of the SABR- Hull White Model for Long- maturity Equity Derivatives" *The journal of computational finance (79-113) volume, 15*, December.

Grzelak, L&Oosterlee, W (2019). "Mathematical Modeling and computation in Finance: With Exercise and Python and Matlab computer codes. ISBN-13: 978-1786347947 https://www.researchgate.net/publication/334748386

Hagan, P&Lesniewaki, A. (2008). LIBOR market model with SABR style volatility. *Jp Morgan Chase and Ellington management Group,32.* https://lesniewski.us/papers/working/SABRLMM

Haksar, V & Kopp, E.(2020). How can Interest Rates be Negatives?. *International Monetary Fund.* https://www.imf.org/en/Publications/fandd/issues/2020/03/what-are-negative-interest-rates-basics

Heston, S. (1993). A closed form for options with stochastic volatility with Applications to Bond and Currency options. The *review of financial studies*, vol. 6, 327-34. https://econpapers.repec.org/article/ouprfinst/v_3a6_3ay_3a1993_3ai_3a2_3ap_3a327-43.htm

Inhoffen, J & Pekanov.A (2021). Low for Long: Side effects of Negative Interest rates. *Monetary dialogue papers,* PE 662.920. https://www.europarl.europa.eu/RegData/etudes/STUD/2021/662920

Mankiw, G. (2009). More on Negative Interest rates. *Greg Mankiw's Blog random observations for students of Economics.* http://gregmankiw.blogspot.com/2009/04/more-on-negative-interest-rates.html

Merton, R. (1973). "Theory of Rational Option Pricing". *The Bell Journal of Economics and Management Science,* Spring, Vol 4, No 1, pp.141-183. https://www.maths.tcd.ie/~dmcgowan/Merton

Mombelli, A. (2020). Who wins and Who Loses Because of Negative interest rates? 45 573032 https://www.swissinfo.ch/eng/swiss-national-bank_who-wins-and-who-loses-from-negative-interest-rates-/45573032

Poitras, C. (2008). The Early History of Option Contracts. *Simon Fraser University*

Robert. F. (2011). Valuing Options in Heston's Stochastic Volatility Model: Analytical Approach. *Journal of Applied Mathematics,* Volume 2011, Article ID 198469, 18 pages, https://downloads.hindawi.com/journals/jam/2011/198469

Tjon, B. (2019). Pricing Derivative in Periods of Low or Negative Interest Rates. Master Thesis. https://thesis.eur.nl/pub/49565

WEBOGRAPHY

1.Dheeraj Vaidya, CFA, FRM. Options Trading. https://www.wallstreetmojo.com/options-trading-strategies. 2021,15th January, 11:00 A.M

2. https://quantpy.com.au/python-for-finance/simulated-stock-portolio April 9 th, 2022

3. Quantstart. Deriving the Black-Scholes Model Equation. https://www.quantstart.com/articles/Deriving-the-Black-Scholes-Equation. December 12th,2021
4. https://github.com/LechGrzelak/Computational-Finance-Course. January 20,2022
5. https://medium.com/quant-guild/market-implied-volatility. February 6th 2022