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# Enhancing Risk Management: Comparing VaR and CVaR Models for Stock Portfolios and Swaptions

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**ABSTRACT**

*This study highlights practical application of Value at Risk (VaR) and Conditional Value at Risk (CVaR) as risk measures for financial portfolios, with a specific focus on stock portfolios and interest products like Swaps and Derivatives. By analyzing calculation methods of CVaR and VaR, this work aims show their effectiveness in capturing tail risk present in different assets classes with varying risk profiles. Through different methods of implementing those two models, relative strengths and limitation of both models will be revealed, offering in return actionable insights for practitioners aiming to optimize risk management strategies with diverse portfolios.*

***Keywords****: VaR, CVaR, Risk management, Parametric method, Historical Method, Monte Carlo Method.*

# Amélioration de la gestion des risques : Comparaison des modèles VaR et CVaR pour des portefeuilles d'actions et des swaptions

**RÉSUMÉ**

*Cette étude met en lumière l'application pratique de la Valeur à Risque (VaR) et de la Valeur à Risque Conditionnelle (CVaR) en tant que mesures de risque pour des portefeuilles financiers, avec un accent particulier sur les portefeuilles d'actions et les produits d'intérêt comme les swaps et les dérivés. En analysant les méthodes de calcul de la CVaR et de la VaR, ce travail vise à démontrer leur efficacité dans la capture des risques de queue présents dans différentes classes d'actifs aux profils de risque variés. À travers diverses méthodes d'implémentation de ces deux modèles, les forces et les limites relatives de chacun seront mises en évidence, offrant en retour des informations exploitables pour les praticiens cherchant à optimiser leurs stratégies de gestion des risques avec des portefeuilles diversifiés.*

***Mots clés****: VaR, CVaR, gestion des risques, méthode paramétrique, méthode historique, méthode de Monte Carlo.*

## INTRODUCTION

In the World of quantitative finance, an efficient risk management method is crucial for ensuring portfolio stability, particularly under market uncertainty where assets are more volatile. Value at risk and conditional value at risk are two of the most commonly used risk measures both of which are integral to modern risk management frameworks. VaR is frequently used to assess the maximum potential loss of a portfolio with a given confidence level over a specific period of time. (Jorion, 2007). Despite its popularity, the value at risk model has limitations especially when dealing with the tail risk of distributions that manifest heavy-tailed or not normally distributed returns. As a matter of fact, Conditional Value at Risk, also called Expected Shortfall, has been adopted by practitioner as a complementary or alternative risk measure that can provide insights into the average loss beyond the VaR threshold offering therefore a more comprehensive view of tail risk (Acerbi&Tasche2002).

The goal of this study is to analyze the implementation steps of the models to two distinct types of portfolios: a traditional portfolio made of stocks and a portfolio consisting of interest rates products. While stock portfolios are subject to equity market volatility, interest rates products present additional levels of complexity due to their sensitivity to interest rates movements, which are often caused by macroeconomic factors, rates policies, and other economic indicators. (Hull,2018). Understanding the determination of VaR and CVaR models is crucial for understanding how risk metrics perform under different conditions and assets classes.

The notion of risk measuring metrics like VaR and CVaR has increased since the financial crisis of 2008, which underscored the relevance of robust risk assessment tools. The crisis exposed the vulnerabilities of traditional risk measures especially when the market behaves unpredictably which motivated banks and regulators to reconsider and reshape existing risk measuring tools for stress testing and scenario analysis (Adrian & Brunnermeier,2016). While VaR is efficient for estimating potential losses in a stable environment, its dependence on a certain confidence level makes it less reliable during periods of high clustered market volatility or portfolios with significant exposure to non-linear instruments such as interest rate derivatives (Embrechts Mc Nel & Staunmann ,2022).

Interest rates products such as swaps, futures and options face unique challenges when determining VaR and CVar because they are not only sensitive to market volatility but also to changes in the yield curve. These products can manifest a wide range of risk profile, with sensitivity to factors like duration and convexity, which impact the efficiency of standard risk measures (Pérignon & Smith 2010). This study will analyze practical implications of using Var and CVaR in portfolios consisting of both stock assets and interest rate products to assess the suitable of these metrics in risk management for diverse asset classes.

## 1. VALUE AT RISK'S MAIN PRINCIPLE

Per definition the Value at Risk is a measure market that provides loss associated with market fluctuations. In practice the model tries to answer the following question: "How much can an investor loose at most on a given investment in a certain period of time?" On a regulatory point of view, central banks require from financial institutions that they keep enough capital to cover potential losses estimated based on VaR methodology. The calculation of VaR results in a quantile connected with a potential loss under the following dynamics:

$$VaR(X) = \inf\{x \in \mathbb{R}: F_X(x) \geq \sigma (1)$$

- $.VaR(X)$ this represents the value at risk of a random variable $X$, which is in most cases a portfolio's loss or return. This metric measures the maximum loss as expected ( but nut exceeded)
- Inf or **Infimum** is the smallest value of x in the set such that the condition that $F_X(x) \geq \sigma$ is satisfied. It essentially identifies the threshold x in scenarios where cumulative probability attains or goes over the confidence level $\sigma$.
- $x \in \mathbb{R}$ the variable x here is a real number and displays potential losses or outcomes.
- $F_X(x)$ represent the **cumulative distribution function (CDF)** of the random variable x specified above.

Depending on the portfolio composition, available market data and the quantile or confidence level $\sigma$, the Value at Risk metric measures the overall risks associated with the market movements.

The main advantage of this measure is its ability to include to one extent the effects of portfolio diversification. The model's features are standardized for capital requirements policies hence banks are required to use a certain confidence level comprises between 95% and 99%. In practice risk managers often require a holding period of 10 days and can consider at least one year of historical data to measure risk factors. Even though parameters are standardized, banks can choose their own approach towards VaR.

Several alternatives have been proposed in order to respond to the lack of coherence of the Value at Risk model. One of the most common is the **Expected Shortfall** or the **Conditional Value at Risk.**

The expected shortfall is defined based on the results of the Value at Risk model's calculation. Its calculation is determined by the following metrics:

$$\mathbb{E}[X|X < VaR_{\sigma}(X)](2)$$

With $VaR(X) = \inf\{x \in \mathbb{R}: F_X(x) \geq \sigma$

When it comes to calculating both the VaR and the CVaR four main methods are used:

- **The parametric Method**: for this particular method, an assumption of normally distributed returns is considered. This method allows only linear portfolios (strong assumptions regarding normality of returns!)
- **Monte Carlo Simulation method**: In this case, a stochastic model is calibrated to historical data and a distribution of portfolio. Through a certain number of Iterations, the Value at Risk model is implemented.
- **Historical Method**: Historical data is used to assess the distribution of the portfolio. VaR model is calculated based on the past stock movement.

**2. Coherent risk measures**

Before analyzing risks associated with capital and potential losses, there is a certain number of principles to be considered for a good risk measure which are the following:

- Sub-additivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$(3). Here $\rho(X)$ is a risk measure applied to a random variable X which usually represents potential financial losses. The parameter $\rho(X+Y)$ represents the combined portfolio (or the total risk of X and Y)
  In this scenario the overall risk of portfolio doesn't exceed the sum of all individual risks. Subadditivity verifies that a portfolio holds the principle of diversification since it always generate a lower risk measure for a diversified portfolio than a non-diversified one.
- Monotonicity:$if\ X \leq Y, \rho(X) \geq \rho(Y)(4)$, In this case $X \leq Y$ the random variable X (representing potential losses) is less riskier than Y in other words if the value of asset X is less or equal of the value of asset Y, this implies that the risk of X should be less than the risk of Y, in simplified terms risks in good assets should be less than those of inferior assets.
- Positive homogeneity: $\rho(aX) = a\rho(X)(5)$, here if the losses (or financial exposure) are scaled up or down by the scaling parameter $a$ , the corresponding risk measure should scale linearly by the same factor $a$
- Transaction invariance:$\rho(X+a) = \rho(X) - a(6)$, under this condition if the cash amount is added to a given asset X, it offsets the corresponding risk associated with X

In some cases the Value at Risk does not satisfy sub-additivity requirement. If a financial institution doesn't abide to sub-additivity requirements, it can encounter some problems. For instance, if a financial implements a VaR without considering the subadditity vity aspect, it is likely to assume too much risk or not hedge when needed.

## 3. SIMULATION ALGORITHM

For any portfolio manager, it is important to identify potential risk that might affect the portfolio return. The following equation is used to express the relationship between value of asset and risk factors: $V(t_0,X), X(t) = [X_1(t), X_2(t) \ldots\ldots, X_n(t)]^T(7)$, where $X_i(t)$ is the risk factor that affects the actual value of a given portfolio. The aim is not to evaluate the portfolio with historical data, but evaluate the current portfolio by taking into consideration historical market movement. Depending on the risk manager, market movements can be considered based on 1 day's increments, 10 days or even longer. Both expected shortfall and value at risk rely on the so-called market scenarios defined simply as increments of the risk factors in time: $\Delta X(s) = X(s) - X(s - \Delta t)(8)$

Historical data, movements are good predictor of future returns especially if one wants to set up a profit and loss profile. $P\&L(t) = V\big(t_0, X(t_0)\big) - V\big(t_0, Xt_0 + \Delta X(S)\big)(9)$,

With $\Delta X(s) = X(s) - X(s - \Delta t)$.

It should be noted that in a portfolio consisting of simple derivatives like spot product, stocks etc.., the calculation of the value at risk calculation is straight forward as only spot values of assets with proper values only needs to be adjusted. However when dealing with interest rate products , the situation becomes much more complicated since every single change in interest rate product would require a re-build of market objects like yield curve.

## 4. CONDITIONAL VALUE AT RISK AND VALUE AT RISK IN PRACTICE

### 4.1. Case of a stock portfolio

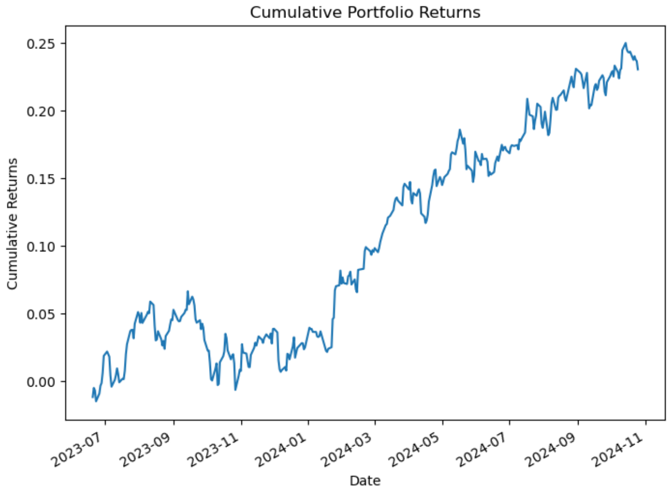
A model combining both historical, parametric and Monte Carlo method is used for this case. And the following steps describe how the model was implemented:

#### a. Data collection

Yfinance library in python is used to fetch historical adjusted stock prices. For this study a portfolio consisting of 5 stocks (Apple, Johnson and Johnson, JPMorgan Chase, Procter &Gamble and Exxon Mobil) was chosen over a defined period of time. From this set of data daily returns, mean returns and covariance matrix to describe the relationship between these assets.

#### b. Portfolio set up

The initial amount of 10000 dollars is set with perfectly balanced weights. During the implementation of the model, weights are however randomly generated and normalized to sum 1. The cumulative returns are plotted to visualize the portfolio's historical growth which can be graphically represented the following way:




### c. Historical VaR and CVaR Calculation.

By considering historical data, the computation of the Value at Risk and Conditional Value at Risk is done. The confidence level for stocks in the initial portfolio is set to 95%. Historical VaR is calibrated to the 5th percentile return (alpha), and the CVaR as the mean of returns below this VaR threshold.

### d. Parametric VaR and CVaR Calculation

For our stock portfolio we assume that the returns follow a normal or t-distribution. Using these two conditions, the risk measures are calculated on the portfolio's expected return and standard deviation. The normal distribution accounts for quantiles of the normal distribution, whereas the t-distribution approach accounts for fat tails with degrees of freedom adjustments.

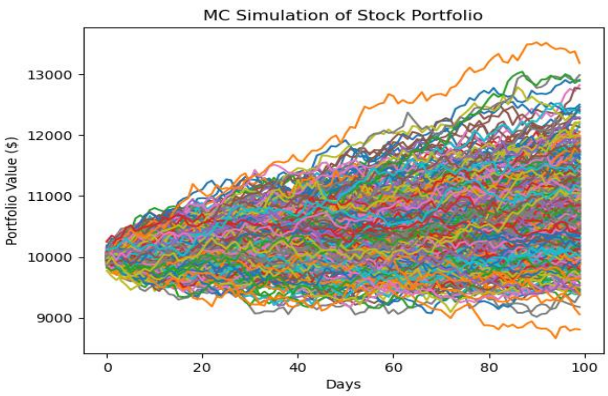
### e. **Monte Carlo Simulation Method**.

In order to get an idea of the portfolio's future value distribution, the Monte Carlo simul-ation m ethod is used. This implies that prices path for each stock are simulated by using a multivariate normal distribution based on mean return and covariance matrix. Each path generates potential portfolio outco mes from which VaR and CVaR under the confidence level of 95% are calculated. The results after bot h methods are combined in one model are the following:

```
Expected Portfolio Return:        $673.94
Historical VaR 95th CI:           $1079.45
Historical CVaR 95th CI:          $1434.14
Normal VaR 95th CI:               $438.12
Normal CVaR 95th CI:              $720.63
t-distribution VaR 95th CI:       $398.74
```

```
t-distribution CVaR 95th CI:     $783.81
Monte Carlo VaR 95th CI:         $350.6
Monte Carlo CVaR 95th CI:        $565.58
```

With the Montecarlo simulation method, it is also possible to plot the potential portfolio values over a time period which provides a representation of possible fluctuations in portfolio value based on simulated returns. The ımage below illustrates the portfolio's possible values on a time scale.




For this study a time frame of 100 days was considered but one can increase or decrease the number of days or path based on desired results.

### 4.2. Interpretation of results

#### a. Expected portfolio return

The expected return of the analyzed portfolio is $673.94 over the investment's time horizon. While this indicates a positive expectation for a portfolio growth, the risk factor should be considered as it can impact the portfolio's return.

#### b. Historical VaR and CVaR

The values of the historical method show that there is a 5% probability that the portfolio could lose at least $1079.45 for the value at the risk and in case the losses exceed the VaR threshold, then our conditional value at risk will be $1434.14. These higher values compared to other methods reflect the worst case scenario provided by the historical market data, more probably capturing the extreme event observed in the past.

#### c. Normal distribution and T-distribution

These methods propose lower values than the historical method. However, for the normal distribution case, suggests that assuming the normality underestimates the actual risk value as it does not consider fat tails or extreme events of returns. The T-distribution on the other hand accounts for fat tails and its approach moderate potential losses which suggests it might be a more conservative and realistic estimate than the normal distribution.

**d. Monte Carlo Simulation**

The Monte Carlo simulation estimates the lowest VaR and CVaR values. This result is obtained after numerous simulated paths rather than relying only historical data or distribution assumptions. However this method might underestimate risk if the simulation does not fully capture extreme market behaviors.

**e. Recommendations**

The higher historical method suggests that our portfolio is sensitive to extreme market scenarios. It is advised to reduce exposure to higher-volatility or reallocating to low-beta assets that didn't historically register high fluctuations. The wide range of VaR estimates across the above methods means that the current diversification might not be fully effective. Considering assets with lower correlations to the our simulated stock portfolio may reduce the overall risk which will therefore lower the historical and simulated VaR and CVaR values. The parametric or normal distribution method minimizes the risk due its principle of normally distributed returns. The T-distribution and the Monte Carlo methods make much more sense for this case since they capture tail events and extreme losses.

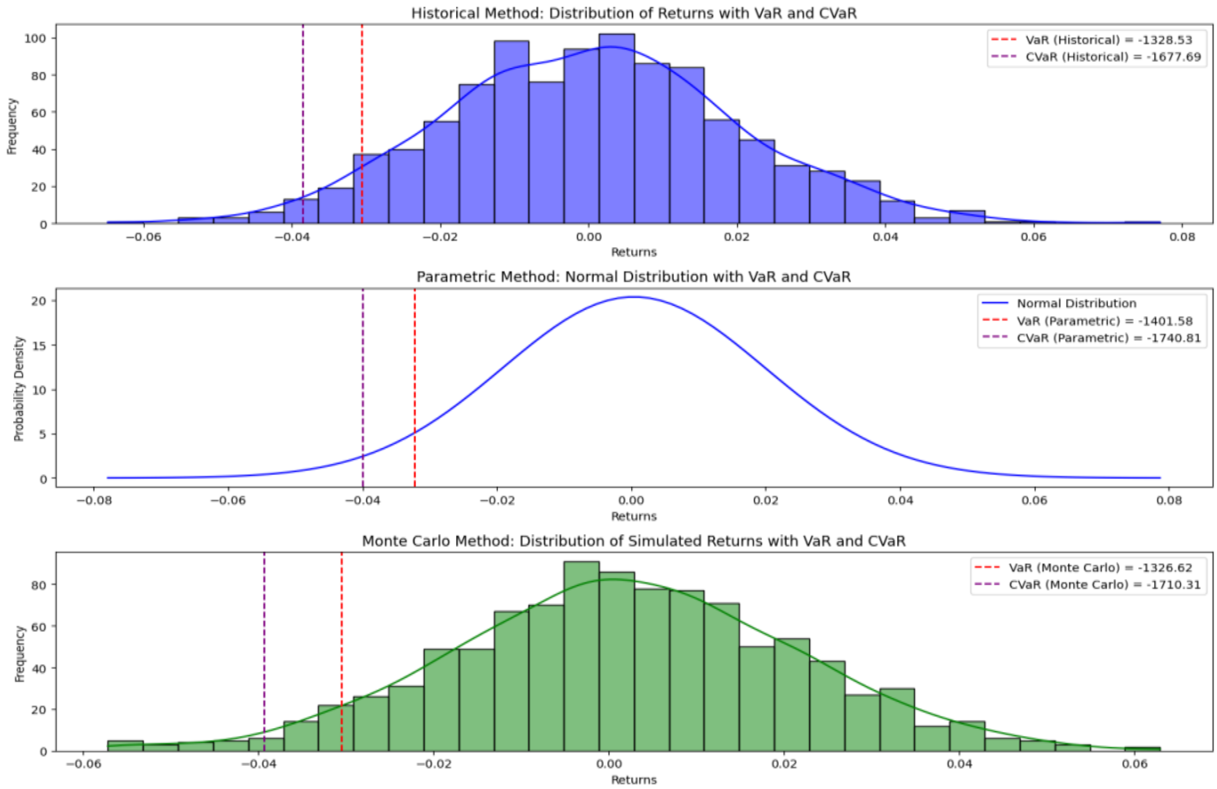
**4.3. The HVaR and CVaR methods applied on a portfolio of interest product**

For this study, we chose randomly a portfolio consisting of swaptions which are by definition option contract that grants the holder the right to enter into a predetermined swap contract. In return, the holder of the swaption must pay a premium to the issuer of the contract.

The components of the swaptions portfolio for the application of the model are the following:

{'notional': 1_000_000, 'fixed_rate': 0.02, 'floating_rate': 0.015, 'maturity': 2y} {'notional': 500_000, 'fixed_rate': 0.025, 'floating_rate': 0.02, 'maturity': 5y}
{'notional': 750_000, 'fixed_rate': 0.03, 'floating_rate': 0.025, 'maturity': 7y}

Since in order to find the value of the portfolio yield curve values must be included, for our model, the data for the annual yield curve are : 1Y=0.015; 2Y=0.017; 3Y=0.018; 5Y= 0.02; 7Y= 0.022 (a sample of yield curve from U.S. Department of treasury)




This figure shows the application of both models (CVaR and VaR) on a portfolio consisting of swaptions. Below is a detailed explanation of how both models perform under the historical, parametric and Monte Carlo methods.

**a. Historical Method (top panel)**

The subplot displays the distribution of historical returns derived from historical data. The histogram (in blue) is the interpretation of the frequency of returns, while the smooth line shows the kernel density estimate to highlight the shape of return distribution's shape.

The vertical red line indicates the maximum expected loss of the VaR model under the confidence level of 95%. The vertical purple line denotes the expected shortfall, which is the measure of the average loss beyond the value at risk threshold, capturing the tail risk. The historical VaR (-1328.53) quantifies the unpredicted loss under 95% confidence level whereas the historical CVaR (-1677.699) displays the average loss in the worst 5% providing a more detailed risk measure. The distribution shows a slightly skewed shape which indicates the relevance of using the CvaR alongside VaR.

**b. Parametric method (middle panel)**

This subplot considers normally distributed returns to estimates the parameters of VaR and CVaR parameters. The blue curve represents the normal probability density function (PDF) according to available data. The red line indicates the parametric VaR (-1401.58), that is determined used the inverse of normal cumulative distribution function (CDF) and the purple line indicates the parametric CVaR (-1740.81), which goes beyond the VaR threshold to estimate expected loss. The advantage of the normal distribution is that it simplifies calculations and provides a quick approximation for risk measures. However this method tends to underestimate risk in non-normal return distributions. The larger conditional value at risk compared to the value at risk emphasizes on the potential severity of extreme losses, especially if the actual return distribution has fat tails.

**c. Monte Carlo method( Bottom panel)**

The subplot presents the distribution returns simulated based on the Monte Carlo method. The histogram (in green), shows the frequency of simulated returns. The red dashed line marks VaR results (-1362.62), derived from the $5^{th}$ percentile of the simulated returns. The purple dashed line displays the Monte Carlo CVaR value (-1710.31), estimated as the average loss in the worst 5% of case scenario.

The Monte Carlo method considers non-normality aspect of returns and includes random scenarios to model risk more flexibly. Its results align closely with historical method which suggests that the historical method used reflects the portfolio's risk characteristics well. This method is particularly handy for complex portfolio or when historical data is insufficient.

**d. Key Insights**

**Risk estimation accuracy:** the choice of a model is crucial for the determination of the accuracy and reliability of VaR and CVaR risk measures. The Historical and Monte Carlo methods are highly recommended for non-normal distributions or portfolios with derivatives like swaptions.

**Tail risk awareness:** CVaR is an important supplement to VaR , as it registers the severity of losses in the tail, providing in return a better understanding of extreme risks.

**Applicability:** The Monte Carlo method is versatile and applicable, making it the ideal model for portfolio with complex risk dynamics, whereas the parametric method can only be applicable normally distributed returns.

## CONCLUSION

This study has elaborated the practical application of value at risk (VaR) and Conditional Value at Risk (CVaR) in managing financial risks across 2 different types of portfolios. By using historical, parametric and Monte Carlo methods, this study highlighted the importance of methodological choice in estimating potential losses under different market conditions.

The finding show that while VaR provides an overview of the maximum expected loss at a certain confidence level, CVaR presents a more comprehensive measure by taking into consideration the tail-end risks since it goes beyond the VaR threshold. Through the approaches used in this study, the Monte Carlo method provides robust insights in scenarios involving complex instruments such as swaptions even though its computation can be intensive.

Moreover, the inclusion of manually defined yield curve and its integration into swaption valuation insures the critical role of accurate input data and realistic assumptions in financial modeling. Overall, the incorporation of VaR and CVaR into portfolio management concept improves the ability of practitioners to make informed decisions, particularly in volatile or uncertain financial environments. Future research could explore how to improve risk estimation and resilience of assets under diverse market conditions by stress testing different market scenario and their impact on financial stability.

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**Retrieve codes used fort his study on** https://github.com/ArmandCharles91/financial-analysis-with-Armand